The COURSE-TIME assignment problem is as follows.

* **Input:** A set of students S, a set of classes C a positive integer K and, for each student *x* ∈ S, a list L of courses that student *x* wants to take.
* **Question:** Is it possible to schedule courses into only K time slots such that each student to take all of his or her desired courses, without any time conflicts?

Prove that the COURSE-TIME assignment problem is NP-complete.

**Hint:** Reduce from the graph colorability problem. (Read about this.) Think of the courses as vertices of a graph, and put an edge between two courses if they cannot be scheduled in the same time slot.

1) The COURSE-TIME problem is in NP.

A solution is an assignment of courses to time slots. To verify the solution, check that each student's list of courses has no time conflicts, and that only the available number of time slots have been used. This can be done in polynomial time.

2) **K-COLOR ≤p** COURSE-TIME

Recall, the graph colorability problem, **K-COLOR,** is as follows.

**Input:** An undirected graph *G* and a positive integer *K*.

**Question:** Is it possible to color the vertices of *G* using no more than *K* colors (coloring each vertex just one color) so that no two adjacent vertices have the same color?

The graph colorability problem reduces to the course assignment problem via reduction defined as follows. Given graph *G* and positive integer *K*, produce a course assignment problem whose courses are the vertices of *G*, and with *K* available time slots. Include, in the problem, a student for each edge of *G*. The student associated with edge (*u*,*v*) wants to take courses *u* and *v*, and nothing more. This reduction can be done in polynomial times assuming there are at most n2 edges, n vertices and n colors.

Suppose that *G* can be colored with *K* colors. Get a *K*-coloring of *G*. Assign times slots to the courses by following the coloring. If vertex *v* is colored by color *m*, then use time slot *m* for course *v*. Since the coloring does not color any two adjacent vertices the same color, there can be no time conflicts.

Suppose that the courses can be assigned time slots so that there are no conflicts. Then assign colors to *G* in the same way, using the *m*-th color for vertex *v* if course *v* received the *m*-th time slot. Since there is a student for each edge, and none of the students have time slot conflicts, this coloring must avoid coloring two adjacent vertices the same color.

**Remark.** Be sure that your reduction goes the right direction. You need to show how to solve the graph coloring problem, assuming that you already have a solution to the course assignment problem, not the other way around.